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**Reg. No. : ...............................**

**Name : ...................................**

# Fourth Semester B.Tech. Degree Examination, February 2016
## (2013 Scheme)
## 13.401 : PROBABILITY, RANDOM PROCESSES AND NUMERICAL TECHNIQUES (FR)

Time : 3 Hours Max. Marks : 100

### PART – A

Answer **all** questions. **Each** question carries 4 marks..

1. If the p d f of a continuous random variable X is given by

$$f(x)=\begin{cases}k(1-x^2), & 0\le x\le 1\\ 0, & \text{otherwise}\end{cases}$$

Find a) the value of k and

b) the probability $P(0.5\le x\le 2)$

2. If X follows Poisson distribution and if $3P(X=2)=2\,P(X=1)$, find $P(X=0)$ and $P(X=3)$.

3. The auto correlation function for a stationary process X(t) is given by $R_X(\tau)=9+2e^{-|\tau|}$. Find the mean value of the random variable $Y=\int_0^2 X(t)dt$ and variance of X(t).

4. Consider a constant random process $X(t)=c$, where c is a random variable with mean $\mu$ and variance $\sigma^2$. Examine whether X(t) is mean ergodic.

5. Find the cubic polynomial which takes the following values :

| X | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| f(x) | 1 | 2 | 1 | 10 |

P.T.O.

PART – B

Answer **one full** question from **each** Module. **Each** question carries **20** marks.

**Module – I**

6. a) Derive the mean and variance of binomial distribution.

   b) In a continuous distribution, the probability density is given by $f(x) = kx(2-x), 0 < x < 2$. Find k, mean, variance and the distribution function.

   c) The time (in hours) required to repair a machine is exponentially distributed with parameter $\lambda = 1/2$.

      i) What is the probability that the repair time exceeds 2 hours ?

      ii) What is the conditional probability that a repair takes at least 10 hours given that its duration exceeds 9 hours ?

7. a) In 800 families with 5 children each, how many would you expect to have i) 3 boys ii) 4 girls iii) at most 2 girls

   b) In a normal distribution 7% of items are below 35 and 80% of items are below 63. What are the mean and standard deviation of the distribution ?

   c) If X is uniformly distributed in [–2, 2] find i) $P(x < 0)$ and ii) $P(|x-1| \geq \frac{1}{2})$.

**Module – II**

8. a) The joint p d f of X and Y is given by $f(x, y) = kxye^{-(x^2+y^2)}, x > 0, y > 0$. Find the value of k and prove that X and Y are independent.

   b) Find the coefficient of correlation from the following data :

| X | 5 | 10 | 15 | 20 | 25 |
|---|---|---|---|---|---|
| Y | 16 | 19 | 23 | 26 | 30 |

   c) The joint probability function of two random variables X and Y is given by $f(x, y) = c(2x + y)$, where x=0, 1, 2 and y = 0, 1, 2, 3 and f(x, y) = 0 otherwise.

      i) Find c and ii) Find $P(X \geq 1, Y \leq 2)$

9. a) The joint density function of random variables X and Y is given by

$f(x, y) = 8xy \quad 0 < x < 1, 0 < y < x$

$= 0 \quad$ else where

Find $P\left(y < \frac{1}{8} / x < \frac{1}{2}\right)$.

b) Given a random variable Y with characteristic function $\varphi(\omega) = E[e^{i\omega Y}]$ and a random process defined by $X(t) = \cos(\lambda t + Y)$, show that $\{x(t)\}$ is stationary in the wide sense if $\varphi(1) = \varphi(2) = 0$.

**Module – III**

10. a) Show that the random process $X(t) = A\cos(\omega_0 t + \theta)$ is wide sense stationary, if A and $\omega_0$ are constants and $\theta$ is uniformly distributed random variable in $(0, 2\pi)$.

b) Find Power spectral density of a WSS process if its auto-correlation function is $R(\tau) = \rho e^{-\rho|\tau|}$.

11. a) A random process is defined as $x(t) = A\cos\omega t + B\sin\omega t$, where A and B are random variables with $E(A)=0$, $E(B)=0$, $E(A^2) = E(B^2)$ and $E(AB) = 0$. Show that the process X(t) is mean ergodic.

b) Find the auto-correlation function of the process {X(t)}, for which the power spectral density is given by

$$S(\omega) = \begin{cases} 1+\omega^2, & |\omega| \le 1 \\ 0, & |\omega| > 1 \end{cases}$$

**Module – IV**

12. a) Find a real root of the equation $x^3 - 2x - 5 = 0$ by Regula-Falsi method.

b) Find $\sqrt[3]{24}$ using Newton-Raphson method.

c) From the following table find f(0.15) and f(0.7).

| X | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 |
|---|---|---|---|---|---|---|
| f(x) | 2.68 | 3.04 | 3.38 | 3.68 | 3.96 | 4.21 |

13. a) Apply Gauss-Seidel method to solve the equations

$$27x + 6y - z = 85,$$
$$x + y + 54z = 110,$$
$$6x + 15y + 2z = 72.$$

b) Given the values

| x | 5 | 7 | 11 | 13 | 17 |
|---|---|---|---|---|---|
| f(x) | 150 | 392 | 1452 | 2366 | 5202 |

Evaluate f(9).

c) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ using

i) Trapezoidal rule

ii) Simpson's 1/3rd rule.